CALCULUS: Graphical, Numerical, Algebraic by Finney, Demana, Watts and Kennedy
Chapter 3: Derivatives Graphs of the Derivative of a function

What you'll Learn About

- How to graph the derivative from the original function
- How to graph the function from the derivative

The graph of a function is given. Choose the answer that represents the graph of its derivative.

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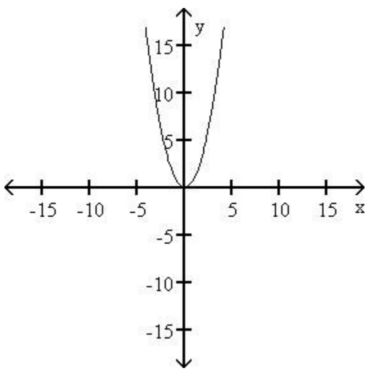

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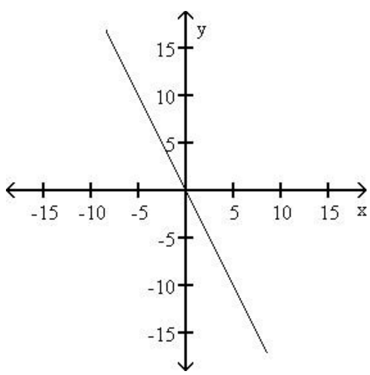

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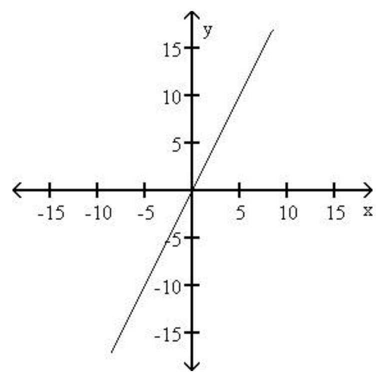

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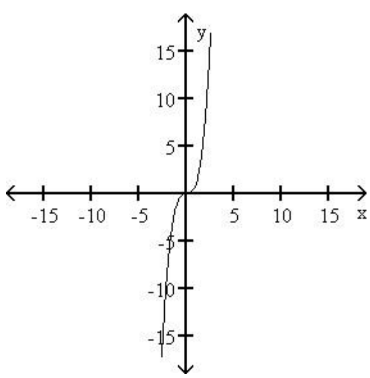

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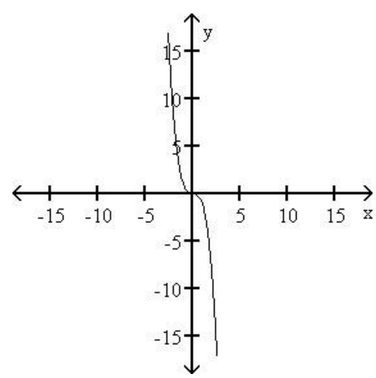

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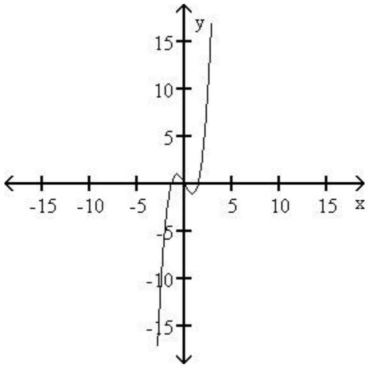
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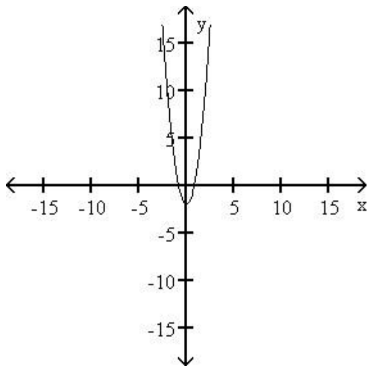
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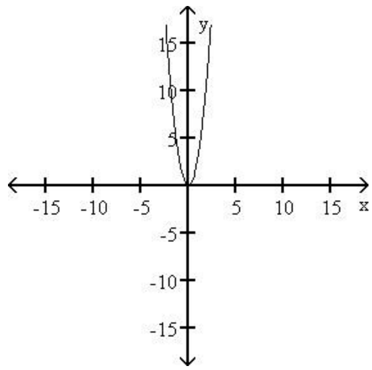
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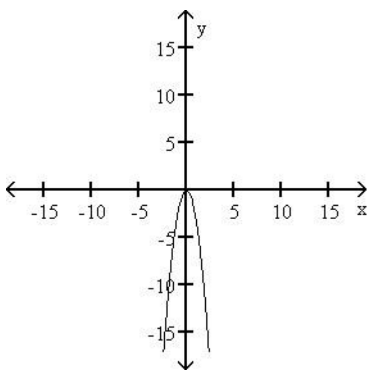
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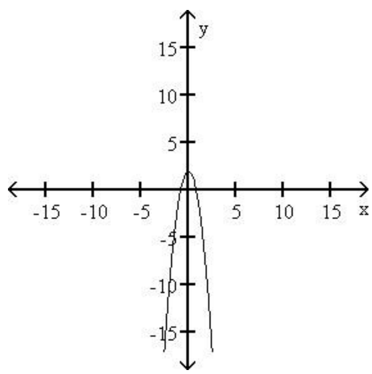
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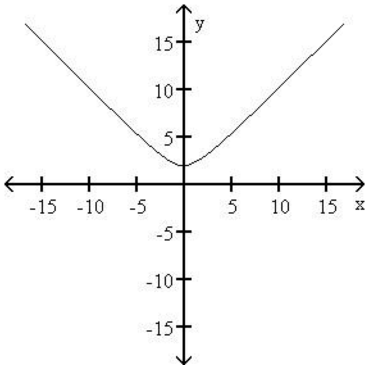
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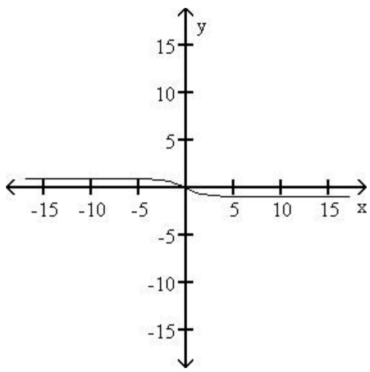
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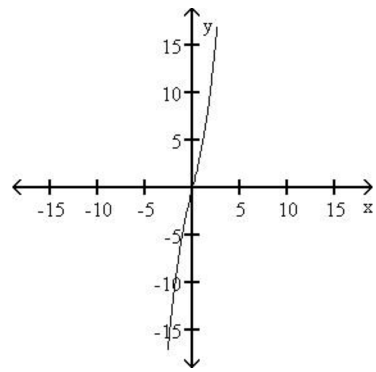
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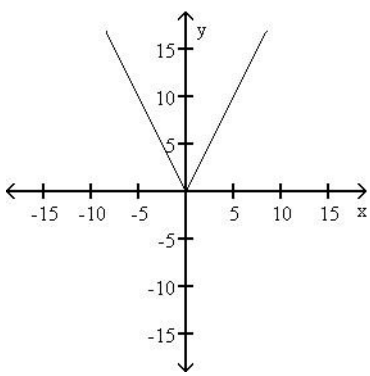
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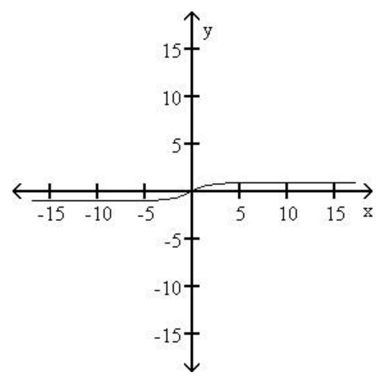
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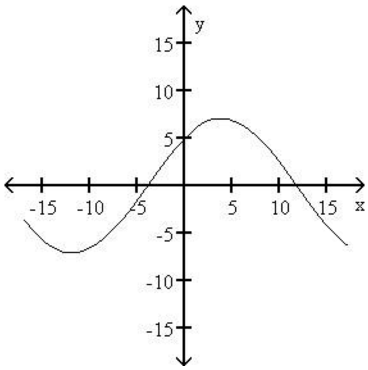

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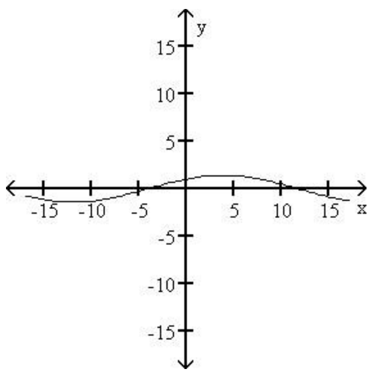

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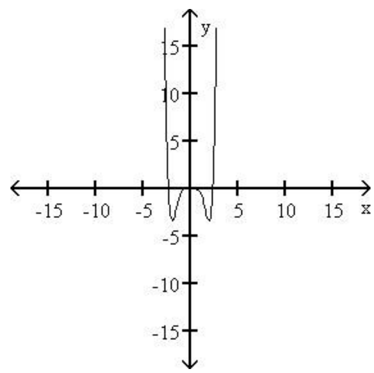

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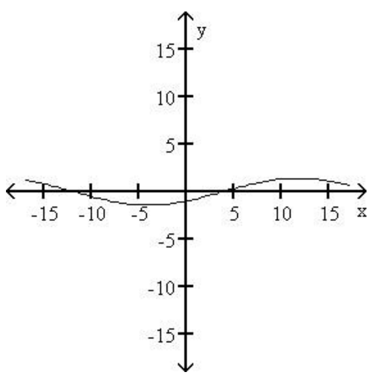

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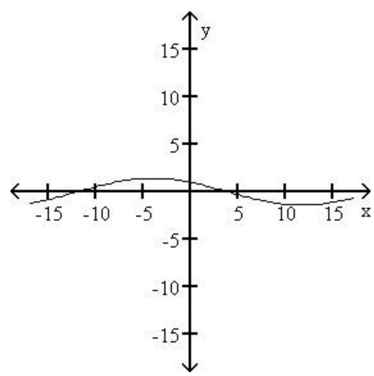

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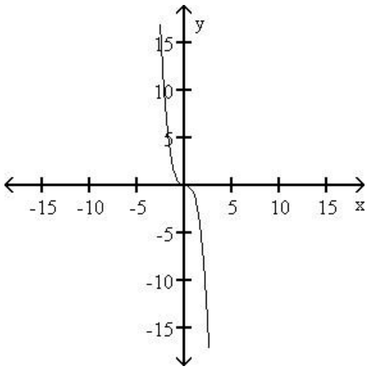
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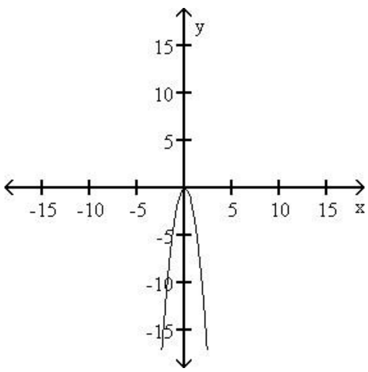
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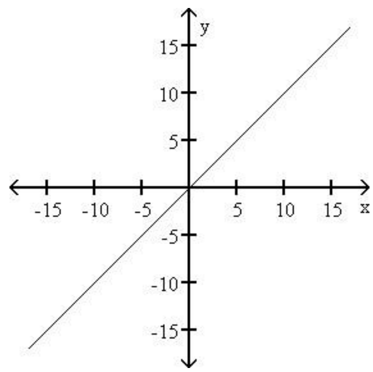
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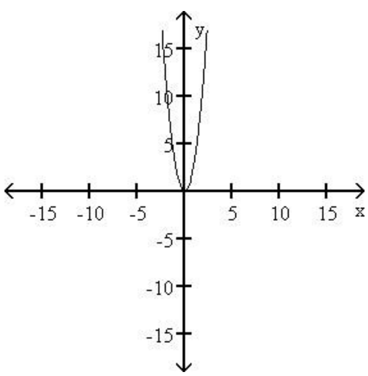
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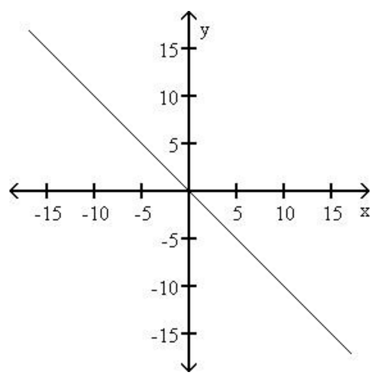
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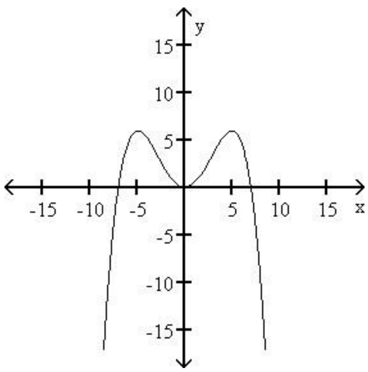
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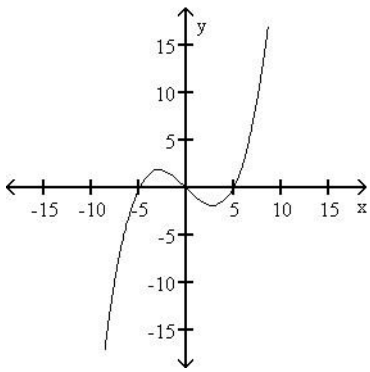
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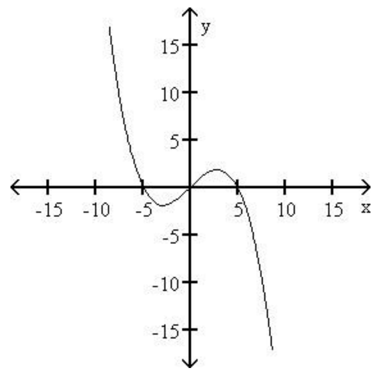
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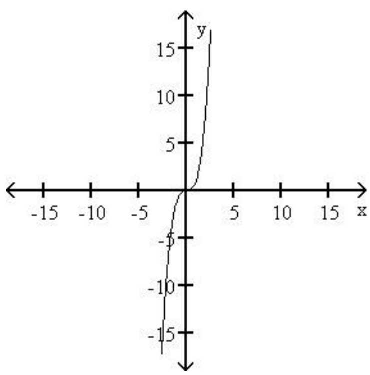
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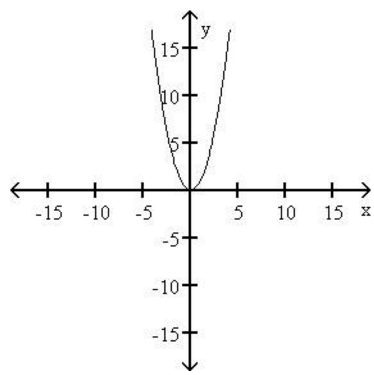
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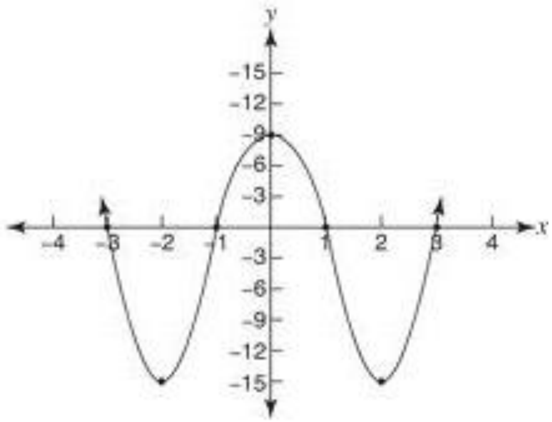
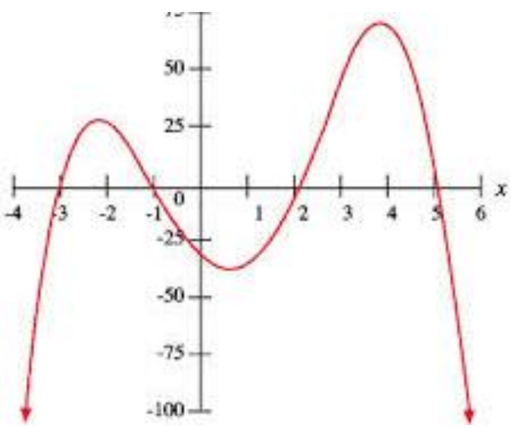
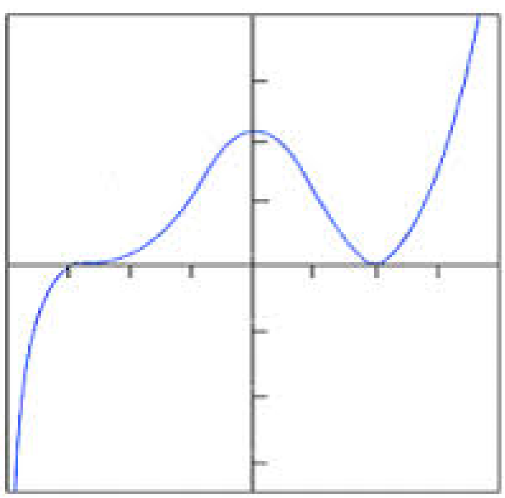
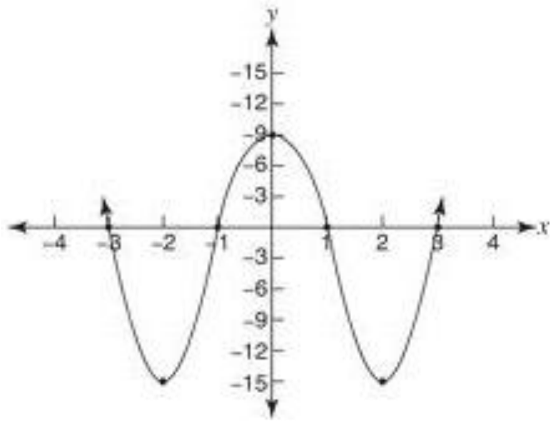
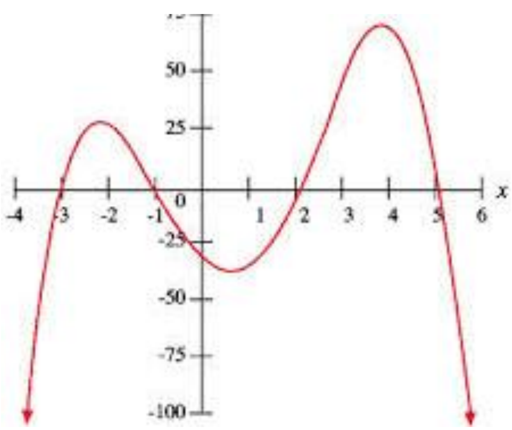
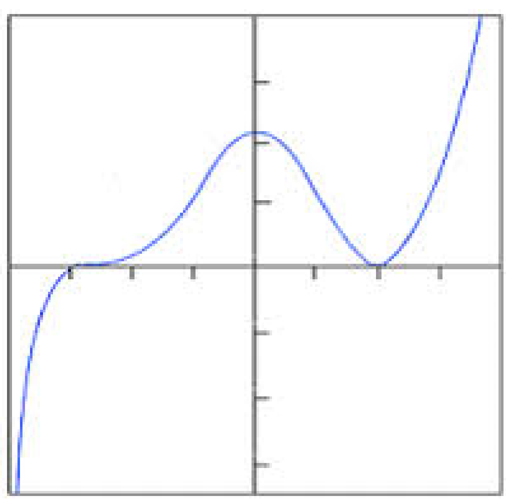
	Sketch $f'(x)$ on the same coordinate plane as the given graph of f(x)   

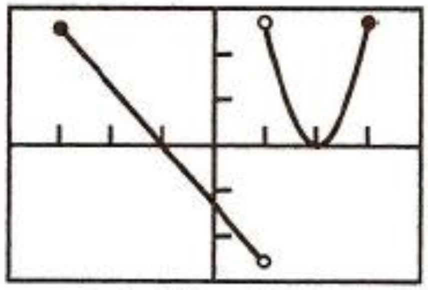
Sketch f(x) on the same coordinate plane as the given graph of $f'(x)$



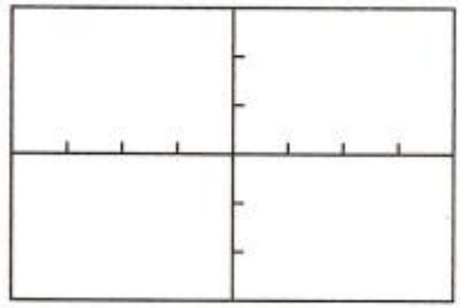




8. Sketch a possible graph of a continuous function f that has domain [-3, 3], where f(-1) = 1 and the graph of y = f'(x) is shown below.



[-4, 4] by [-3, 3]



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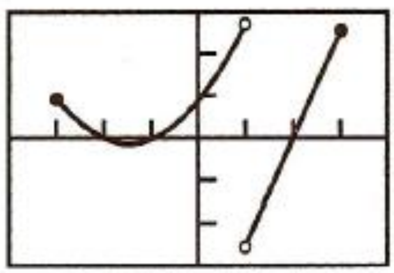
9 Sketch a possible graph of a continuous function f that has domain [-3, 3], where f(-1) =- 1 and the graph of y = f'(x) is shown below.



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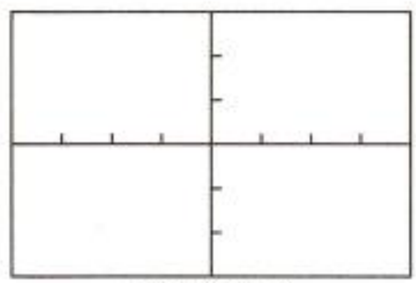
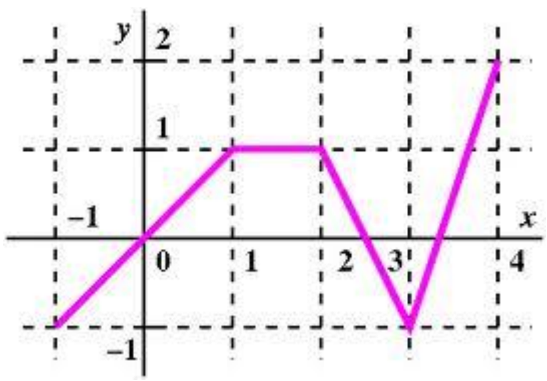


[-4, 4] by [-3, 3]

p. 107 #27 Sketch the graph of a continuous function f with f(0) = -1 and

$$f'(x) = \begin{cases} 1, & x < -1 \\ -2, & x > -1 \end{cases}$$

The graph of the function f(x) is shown here is made of line segments joined at each end.



a. Graph the functions derivative.

b. At what values of x between x = -1 and x = 4 is the function not differentiable?

What you'll Learn About
How to find the derivative at a point given a table of values

2013 BC3

Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t, $0 \le t \le 6$, is given by a differentiable function C, where t is measured in minutes. Selected values of C(t), measured in ounces, are given in the table.

t(minutes)	0	1	2	3	4	5	6
C(t) ounces	0	5.3	8.8	11.2	12.8	13.8	14.5

a) Use the data in the table to approximate $C'(5.5)$. Show the computations that lead to your answer, and indicate units of measure.

2011 #2

t(minutes)	0	2	5	9	10
H(t) degrees C	66	60	52	44	43

As a pot of tea cools, the temperature of the tea is modeled by a differentiable function H for $0 \le t \le 10$, where time t is measured in minutes and temperature H(t) is measured in degrees Celsius. Values of H(t) at selected values of time t are shown in the table above

Use the data in the table to approximate the rate at which the temperature of the tea is changing at time t = 9.5. Show the computations that lead to your answer.

2012 #1

t(minutes)	0	4	9	15	20
W(t) degrees F	55.0	57.1	61.8	67.9	71.0

The temperature of water in a tub at time t is modeled by a strictly increasing, twice differentiable function, W, where W(t) is measured in degrees Fahrenheit and t is measured in minutes. At time t = 0, the temperature of the water is 55° F. The water is heated for 30 minutes, beginning at time t = 0. Values of W(t) at selected times t for the first 20 minutes are given in the table above.

a) Use the data in the table to estimate $W'(17.5)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

2010 #2

A zoo sponsored a one-day contest to name a new baby elephant. Zoo visitors deposited entries in a special box between noon (t=0) and 8 P.M. (t=8). The number of entries in the box t hours after noon is modeled by a differentiable function E for $0 \le t \le 8$. Values of E(T), in hundreds of entries, at various times t are shown in the table.

t(hours)	0	2	5	7	8
E(t) (hundreds of entries)	0	4	13	21	23

b) Use the data in the table to approximate the rate in hundreds of entries per hour, at which entries were being deposited at time t = 7.5. Show the computations that lead to your answer.

2016 BC 1

t(hours)	0	1	3	6	8
R(t) liters/hour	1340	1190	950	740	700

Water is pumped into a tank at a rate modeled by $W(t) = 200e^{-t^2/20}$ liters per hour for $0 \le t \le 8$, where t is measured in hours. Water is removed from the tank at a rate modeled by R(t) liters per hour, where R is differentiable and decreasing on $0 \le t \le 8$. Selected values of R(t) are shown in the table above. At time t = 0, there are 50,000 liters of water in the tank.

a) Estimate $R'(2)$. Show the work that leads to your answer. Indicate units of measure.

d) For $0 \le t \le 8$, is there a time when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank. Explain why or why not?

2012 #4

The function f is twice differentiable for x > 0 with f(1.2) = 5 and $f''(1) = 20$. Values f', the derivative of f, are given for selected values of x in the table.

x	1	1.1	1.2	1.3	1.4
$f'(x)$	8	10	12	13	14.5

a) Write an equation for the line tangent to the graph of f at x = 1.2. Use this line to approximate f(1.4).

What you'll Learn About
The derivative represents velocity
The second derivative represents acceleration

13a) Lunar Projectile Motion: A rock thrown vertically upward from the surface of the moon at a velocity of 20 m/sec reaches a height of $s = 20t - .8t^2$ in t seconds.

a) Find the rock's velocity and acceleration as functions of time.

b) How long did it take the rock to reach its highest point?

c) When did the rock reach half its maximum height?

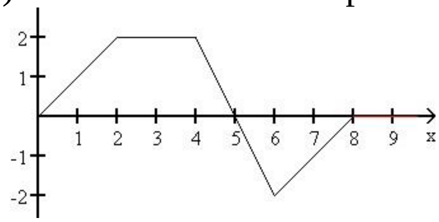
d) How long was the rock aloft?

	p. 137 (19) A particle moves along a line so that its position at any time $t \geq 0$ is given by the function s(t) = $t^2 - 3t + 2$ where s is measured in meters and t is measured in seconds. a) Find the displacement during the first 5 seconds. b) Find the average velocity during the first 5 seconds. c) Find the instantaneous velocity when t = 4. d) Find the acceleration of the particle when t = 4. e) At what values of t does the particle change direction? f) Describe the particles motion

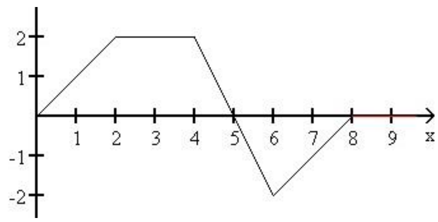
	a) Find the body's velocity, speed, and acceleration at time t. b) Find the the body's velocity, speed, and acceleration at time $t = \frac{\pi}{4}$ 15. s(t) = 2sint + 3 cost

In each situation below, the graph given is the graph of the velocity function

a) Determine when the particle is moving forward and moving backward

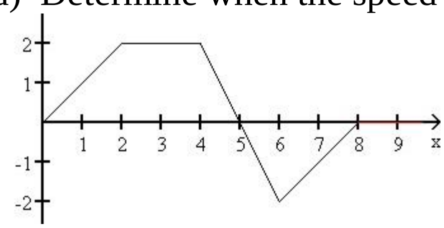


b) Determine when the acceleration of the particle is positive, negative, and zero.

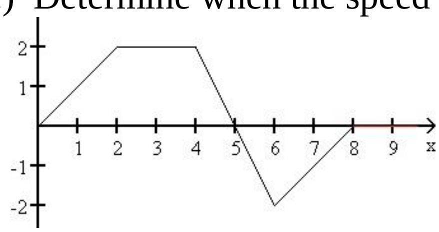


c) Determine when the particle is at its greatest speed.

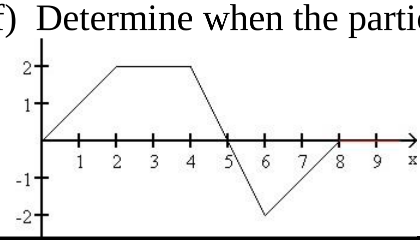
d) Determine when the speed is increasing.



e) Determine when the speed is decreasing.

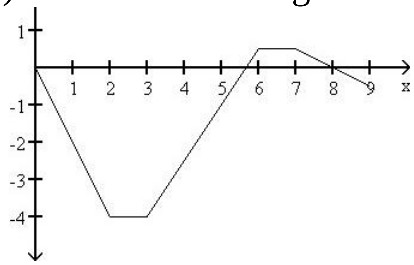


f) Determine when the particle is standing still.

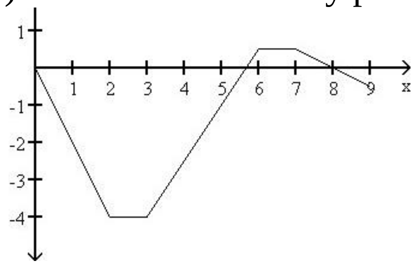


In each situation below, the graph given is the graph of the position function

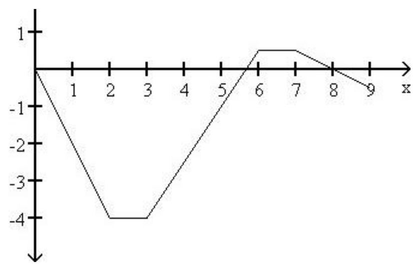
a) When is P moving to the left, to the right, and standing still?



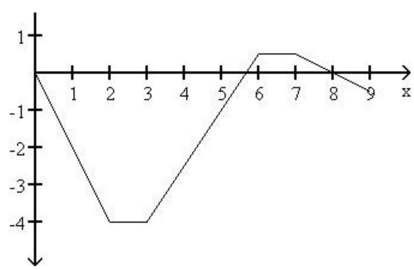
b) When is the velocity positive, negative, and zero



c) Graph the particles velocity



d) Graph the particles speed



Particle Motion Summary Given the **Velocity v(t)** graph

Determine when the particle	Justify/Explain/Give a reason	Where to look on the velocity graph
Forward/Up/Right	$v(t) > 0$	Above the x-axis
Backward/Down/Left	$v(t) < 0$	Below the x-axis
Stopped/At rest	$v(t) = 0$	Touches x-axis
Changes Direction	$v(t) = 0$ and $v(t)$ changes sign	Crosses x-axis
Acceleration Positive	$v'(t) > 0$	Positive slope/Increasing
Acceleration Negative	$v'(t) < 0$	Negative slope/Decreasing
Acceleration Zero	$v'(t) = 0$	Zero slope/Constant
Acceleration Undefined	$v'(t)$ undefined	Corners/Cusps/Vertical Tangents
Speed increasing Speeding up	$v(t)$ and $a(t)$ have the same sign	Graph moving away from the x-axis
Speed decreasing	$v(t)$ and $a(t)$ have opposite signs	Graph moving toward the x-axis

What you'll Learn About

How to take the derivative of a function that is not solved for y (an implicitly defined function)

Find the derivative of the following function

A) $x^2 + y^2 = 1$

B) $x = \cos\theta \quad y = \sin\theta$

C) $x^2 + y^2 = 1$

D) $x^2 + y^2 = xy$

	$E) \quad x^2 = \dfrac{x-y}{x+y}$ $F) \quad x + \tan(xy) = y$

	Determine the slope of the function at the given value of x $G)\ (x+2)^2+(y+3)^2=25$ Find where the slope of the curve is undefined $H)\ x^2+4xy+4y^2-3x=6$

	Find the lines that are tangent and normal to the curve at the given point $I)\ x^2 - \sqrt{3}xy + 2y^2 = 5 \qquad (\sqrt{3},2)$ Find the lines that are tangent and normal to the curve at the given point $J)\ x\sin(2y) = y\cos(2x) \qquad \left(\frac{\pi}{4},\frac{\pi}{2}\right)$

	Determine the 2nd derivative of the function defined implicitly $K)\ \ 2x^3 - 3y^2 = 8$ $L)\ \ x^{\frac{1}{3}} - y^{\frac{1}{3}} = 1$

	10. Consider the curve defined by the equation $x^2 + xy + y^2 = 27$ a) Write an expression for the slope of the curve at any point (x, y). b) Find the points on the curve where the lines tangent to the curve are vertical. c) Find $\frac{d^2y}{dx^2}$ in terms of y.

	Consider the curve defined by the equation $2y^3 + 6x^2y - 12x^2 + 6y = 1$ with $\frac{dy}{dx} = \frac{4x - 2xy}{x^2 + y^2 + 1}$ b) Write an equation of each horizontal tangent to the curve c) The line through the origin with slope -1 is tangent to the curve at point P. Find the x and y-coordinates of P. d) Find $\frac{d^2y}{dx^2}$ in terms of y.

CALCULUS: Graphical, Numerical, Algebraic by Finney, Demana, Watts and Kennedy

Chapter 4: Applications of Derivatives

4.2/4.5: Linearization/Differentials/Mean Value Theorem pg. 196-204

What you'll Learn About
Linearization is another term for tangent line
Differentials are part of the derivative
Mean Value Theorem

a) Find the linearization of the function. b) Find L(a + .1) and f(a + .1)
c) Using concavity, determine if the Tangent Line at a is an overestimate or an underestimate. Justify your answer.

2. $f(x) = x^2 - 2x + 3 \quad a = 2$

1. $f(x) = \sqrt{1+x} \quad a = 0$

	Find dy and evaluate dy for the given value of x and dx 20) $y = \frac{2x}{1+x^2}$ $x = -2$ and $dx = .1$ 24) $y = 3\csc\left(1 - \frac{x}{3}\right)$ $x = 1$ and $dx = .1$

	Use the Mean Value Theorem to determine where the slope of the secant line equals the slope of the tangent line A) $f(x) = x^2$ [2,4] B) $f(x) = x^{\frac{1}{3}}$ [1,8] C) $f(x) = x^{\frac{1}{3}}$ [0,1] D) $f(x) = x^2$ [-2,2]

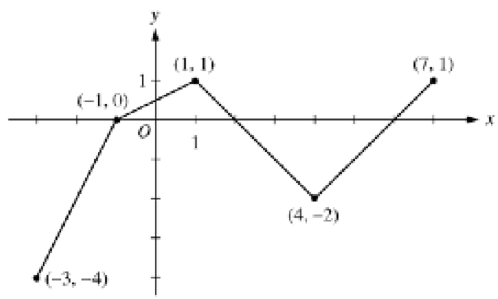
2013 BC3

Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t, $0 \le t \le 6$, is given by a differentiable function C, where t is measured in minutes. Selected values of C(t), measured in ounces, are given in the table.

t(minutes)	0	1	2	3	4	5	6
C(t) ounces	0	5.3	8.8	11.2	12.8	13.8	14.2

Is there a time t, $3 \le t \le 6$, at which $C'(t) = 1$. Justify your answer.

Let g be a continuous function with g(2) = 5. The graph of the piecewise-linear function g', the derivative of g , is shown for $-3 \le x \le 7$.

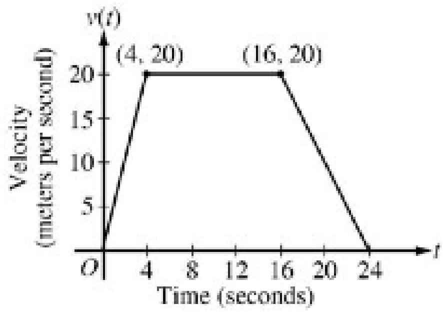


Graph of g'

Find the average rate of change of $g'(x)$, on the interval $-3 \le x \le 1$. Does the Mean Value Theorem applied on the interval $-3 \le x \le 1$ guarantee a value of c, for -3 < c < 1 , such that g''(c) is equal to this average rate of change? Why or why not?

2005 AB5

A car is traveling on a straight road. For $8 \le t \le 24$ seconds, the car's velocity v(t), in meters per second, is modeled by the piecewise-linear function defined by the graph



Find the average rate of change of v over the interval $0 \le t \le 16$. Does the Mean Value guarantee a value of c, for 0 < c < 16, such that $v'(t)$ is equal to this average rate of change? Why of why not?

2004 BCB3

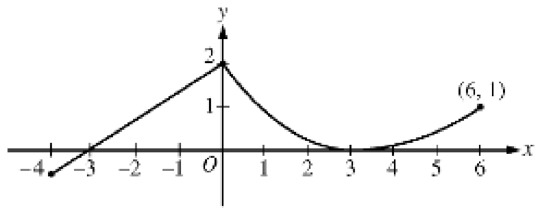
A test plane flies in a straight line with positive velocity v(t), in miles per minute at time t minutes, where v is a differentiable function of t. Selected values of v(t) are shown.

t(min)	0	5	10	15	20	25	30	35	40
v(t) (mpm)	7	9.2	9.5	9.2	4.5	2.4	4.5	4.9	7.3

Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval 0 < t < 40? Justify your answer

2009 BC3

A continuous function f is defined on the closed interval $-4 \le x \le 6$. The graph of f consists of a line segment and a curve that is tangent to the x-axis at x = 3, as shown in the figure above. On the interval 0 < x < 6, the function f is twice differentiable, with $f''(x) > 0$.



Graph of f

Is there a value a, for which the Mean Value Theorem, applied to the interval [a, 6], guarantees a value c, a < c < 6, at which $f'(c) = \frac{-1}{6}$? Justify your answer.

2011 BCB5

Ben rides a unicycle back and forth along a straight east-west track. The twice-differentiable function B models Ben's position of the track, measured in meters from the western end of the track, at time t, measured in seconds from the start of the ride. The table gives values of B(t) and Ben's velocity, v(t), measured in meters per second, at selected times t.

t(seconds)	0	15	40	60
B(t) (meters)	100	136	9	46
V(t) meters per second	2	2.3	2.5	4.6

For $15 \le t \le 60$, must there be a time t when Ben's velocity is -2 meters per second? Justify your answer.

	92. Let f be the function defined by f(x) = x + ln(x). What is the value of c for which the instantaneous rate of change of f at x = c is the same as the average rate of change of f over [2, 6]? If $f(x) = \cos\left(\frac{x}{2}\right)$, then there exists a number c in the interval $\frac{\pi}{2} < x < \frac{3\pi}{2}$ that satisfies the conclusion of the Mean Value Theorem. Find those values.

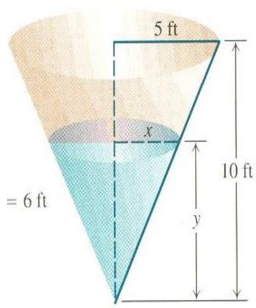
What you'll Learn About
How to use derivatives to solve a problem involving rates

A) Water is draining from a cylindrical tank with radius of 15 cm at 3000 cm^3/second. How fast is the surface dropping?

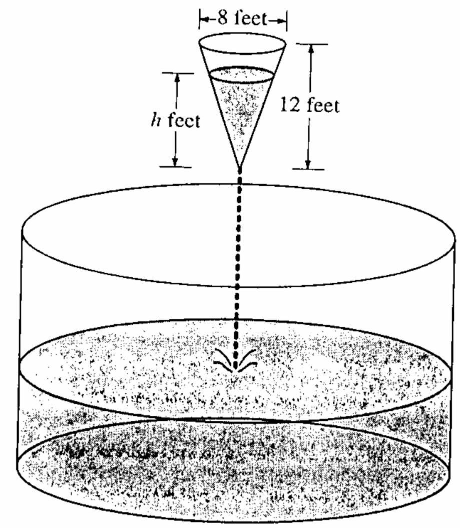
B) A hot-air balloon rising straight up from a level field is tracked by a range finder 500 ft from the lift-off point. At the moment the range finder's elevation angle is 45°, the angle is increasing at the rate of .14 rad/min.
How fast is the balloon rising at that moment?

C) Truck A travels east at 40 mi/hr. Truck B travels north at 30 mi/hr. How fast is the distance between the trucks changing 6 minutes later?

D) Water runs into a conical tank at the rate of 9 ft^3/min. The tank stands point down and has a height of 10 ft and a base radius of 5 ft. How fast is the water level rising when the water is 6 ft deep?



21. Water is draining from a conical tank with height 12 feet and diameter 8 feet into a cylindrical tank that has a base with area 400π square feet. The depth, h, in feet, of the water in the conical tank is changing at the rate of (h - 12) feet per minute. Volume of a cone: $V = \frac{1}{3}\pi r^2 h$



A) Write an expression for the volume of water in the conical tank as a function of h.

B) At what rate is the volume of water in the conical tank changing when h = 3? Indicate units of measure.

C) Let y be the depth, in feet, of the water in the cylindrical tank. At what rate is y changing when h = 3? Indicate units of measure.

What you'll Learn About:
How to use derivatives to find limits in an indeterminate form

Why L'Hopitals Works

Sketch the graph of two curves with the following characteristic f(2) = g(2) = 0.

a) Write the tangent line for f(x)

b) Write the tangent line for g(x)

c) $\lim_{x \to 2} \frac{f(x)}{g(x)}$

d) $\lim_{x \to 0} \frac{2x^2}{x^2}$

2) $\lim_{x \to 0} \frac{\sin(5x)}{x}$

4) $\lim\limits_{x\to 1} \dfrac{\sqrt[3]{x}-1}{x-1}$

49) $\lim\limits_{x\to 1} \dfrac{x^3-1}{4x^3-x-3}$

A) $\lim\limits_{x\to \infty} \dfrac{x^3-1}{4x^3-x-3}$

27) $\lim\limits_{x\to \infty} \dfrac{\ln\left(x^5\right)}{x}$

35) $\lim\limits_{x\to \infty} \dfrac{\log_2(x)}{\log_3(x+3)}$

33) $\lim\limits_{x\to 0} \dfrac{\sin\left(x^2\right)}{x}$